AC Waveforms and Measurements Review

This supplement provides a review of basic ac waveforms and measurements. These concepts apply to both sine waves and more complex waveforms. Power measurements of modulated waveforms are also discussed. This material is covered in both the General and Amateur Extra class exams. For more information, see the *ARRL Handbook* or online ARRL Technical Information Service (**www.arrl.org/tis**). A math supplement is also provided on the web page for the General and Extra Class manuals.

Before we study specific terms and measurements associated with ac signals and waveforms, it's a good idea to review the basic concepts. Students familiar with basic ac waveforms can skip this first section but should check the related exam questions to be sure of their answers. We'll begin by studying the sine wave in depth.

TYPES OF WAVEFORMS

Sine Waves

Not only is a sine wave the most fundamental ac waveform — energy at a single frequency — it also describes rotating motion. Imagine painting a dot on the circumference (rim) of a wheel. Spin the wheel at a constant rate and watch that point!

Figure 1 illustrates how this works. If you watch the wheel on-edge as in Figure 1A, the dot will just move up and down. If we designate the height at point C as +1 and at point G as –1, then make a table of values as the wheel rotates through 360°, the height values will correspond exactly to the values of the sine function (*sin*). When the wheel is at 0° in position A, sin (0°) = 0; when at 90° in position C, sin (90°) = 1, and so forth all the way around.

Now look at the wheel from the side as in Figure 1B. Plotting the dot's height around its circular path against degrees on the horizontal axis will then trace out a sine wave as in Figure 1C.

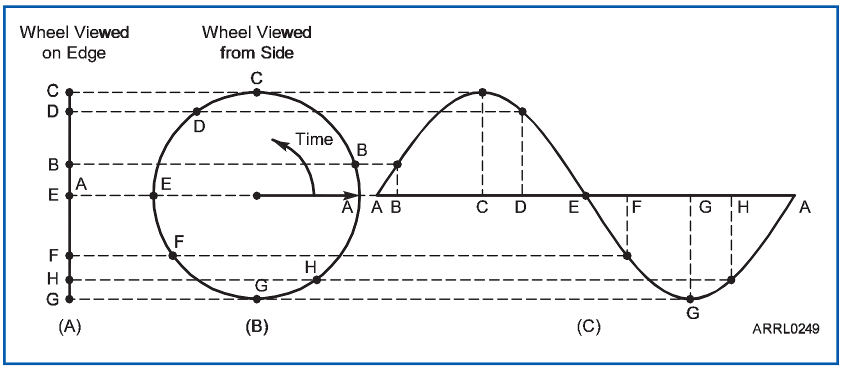

Figure 1 — This diagram illustrates the relationship between a sine wave and circular rotation. You can see how various points on the circle correspond to values on the sine wave.

Each rotation of the wheel corresponds to one *cycle* of the sine wave. The amplitude (A) of the sine wave is equal to the sine of the wheel's *angular position* in degrees (θ):

$$A = \sin(\theta) \qquad \text{(Equation 1)}$$

In the sine wave, angular position is referred to as *phase*.

In Figure 1B, the arrow drawn from the center of the wheel to point A is a *vector*. The vector in Figure 1B has an amplitude equal to its length, which we arbitrarily decided would be 1 when constructing the table of sine values. Since the vector is pointing exactly along the horizontal axis, its direction is 0°. Thus, the vector is described as "1 at an angle of 0°" or "1 with a phase of 0°." In the *phasor notation* commonly used in electronics, this is written $1\angle 0°$. (Vectors and phasors are introduced in the Components and Building Blocks chapter of the Extra Class License Manual.) The rotation of the wheel and the repeated cycles of the sine wave can also be described by a vector that is rotating (spinning around like the hand of a clock) at the frequency of the sine wave.

Now let's make the connection between angle and time. If the sine wave has a constant frequency, every complete cycle takes the same amount of time (the *period*, T) and each degree of phase represents the same amount of time. For example, a sine wave with a frequency of four cycles per second has a period T = 0.25 second and each degree of phase is equivalent to T/360 = 0.25 / 360 = 0.00069 second.

To be able to use Equation 1 to calculate the amplitude, A, of the sine wave at any point in time, t, we need to be able to convert time to the angle, θ:

$$\theta = 360\frac{t}{T} \qquad \text{(Equation 2)}$$

The sine wave equation is now:

$$A = \sin\left(360\frac{t}{T}\right) = \sin\left(360 \times \frac{1}{T} \times t\right) = \sin(360 \times f \times t) \qquad \text{(Equation 3)}$$

In many technical fields the mathematics works out better if *radians* are used instead of degrees. There are 2π (6.28) radians in a circle, so 1 radian = 360 / 2π = 57.3°. Because this is so common, the symbol ω is used instead, with $\omega = 2\pi f$. ω is called *angular frequency*. Equation 2 is then written:

$$A = \sin(2\pi ft) = \sin(\omega t)$$

The license exam and ARRL license manuals always use f, but you will encounter ω in technical references and articles.

Complex Waveforms

A signal composed of more than one sine wave is called a *complex waveform*. A simple example of a complex waveform is used by telephones when dialing. This waveform is composed of two different sine wave tones, thus the name "dual-tone multi-frequency" or DTMF for that signaling system. Listen carefully next time you dial and you will hear the two tones of different frequencies.

There are certain well-known and common complex waveforms that are made up of a sine wave and its harmonics. These are called *regular* waveforms because the harmonic relationship of all the sine waves results in a waveform with a single overall frequency and period. The lowest frequency sine wave in a regular waveform is called the *fundamental*. A waveform that is made of sine waves that are not harmonically related, such as human speech, is an *irregular* waveform. Whether regular or irregular, the sine waves that make up a complex waveform are called its *components*.

To be able to discuss complex waveforms more easily, it is useful to be able to make a graph of all the components by frequency and amplitude as in **Figure 2**. This is called a *frequency domain* or *spectrum* graph. Each component of the signal is shown as a vertical line, with the height of the line showing the component's amplitude. Note that each component occupies a single frequency. Which harmonics are combined with the fundamental and the relative amplitude of each component determine the final shape of the waveform as you will see in the following two sections. The set of all components that make up a signal is called the signal's *spectrum*. (More than one spectrum is *spectra*.)

AC MEASUREMENTS

Because an ac signal's *instantaneous* voltage and current change from one instant to the next, how do you measure it? Is there a single point at which the measurement is taken? Since a sine wave is positive and negative for exactly the same amount of time, shouldn't the value be zero? Actually, the average dc voltage and current in a sine wave ac signal are zero! A dc meter connected to a sine-wave ac voltage will read zero, too. Obviously, ac signals do deliver power and so other measurements than averaging are used to measure the value of ac voltage and current.

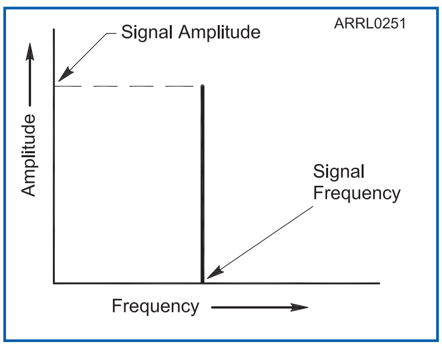

Figure 2 — A frequency domain or spectrum graph shows a sine wave as a single vertical line. The horizontal axis represents frequency and the vertical axis represents amplitude. The height of the line representing the sine wave shows its amplitude.

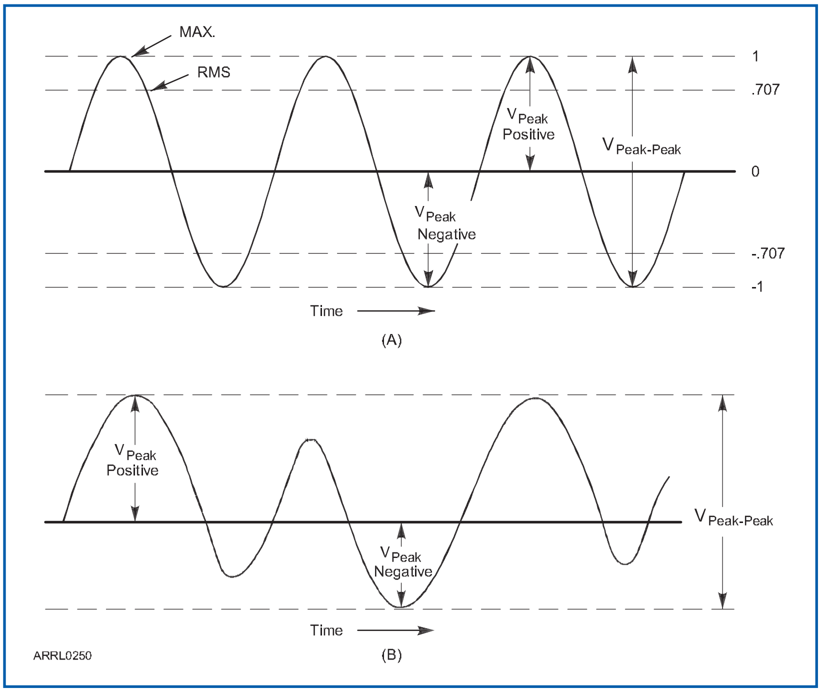

Figure 3 — The various parameters of a sine wave are shown at A. At B, an asymmetric, complex waveform is shown whose positive peak voltage and negative peak voltage are different.

Table 1
AC Measurements for Sine and Square Waves

	Sine Wave	***Square Wave***
Peak-to-Peak	2 × Peak	2 × Peak
Peak	0.5 × Peak-to-Peak	0.5 × Peak-to-Peak
RMS	0.707 × Peak	Peak
Peak	1.414 × RMS	RMS
Average	0 (full cycle)	0 (full cycle)
	0.637 × Peak (half cycle)	0.5 × Peak (half cycle)

If you look at the sine wave of Figure 1, the easiest dimension to measure is the vertical height of the waveform. The maximum positive or negative voltage is called the *peak voltage,* V_{Peak} or V_P. The voltage from the maximum positive to the maximum negative peak is called the *peak-to-peak* value, $V_{P\text{-}P}$ or $V_{Pk\text{-}Pk}$. (From here on, we'll assume the ac waveforms represent voltage, unless specifically noted otherwise.) In a symmetrical waveform, peak voltage has half the value of the peak-to-peak voltage. **Figure 3** illustrates the various voltage parameters of a sine wave.

For common symmetric ac waveforms the conversions between peak, peak-to-peak, average and RMS are simple. **Table 1** shows how to convert between peak, peak-to-peak, average and

RMS waveforms of sine and square waves. You will make frequent use of the sine wave conversions.

It is important to know the peak value of the voltage when picking an electronic component to be used with that waveform. For example, suppose you wanted to install a capacitor across a 48 V transformer output to get rid of some interference. If you used a capacitor rated for 50 V, it would likely fail because the peak voltage from the transformer is actually 67 V!

AC Power

The terms RMS, average and peak have different meanings when referring to ac power. When the sine waves for voltage and current are in phase such as in a resistor, power is the *product* of RMS voltage and current. In this case, RMS and average power are the same, so only average power, P_{AVG}, is used.

$$V_{RMS} \times I_{RMS} = P_{AVG} \qquad \text{(Equation 4)}$$

For continuous sine waves with voltage and current in phase:

$$V_{PEAK} \times I_{PEAK} = P_{PEAK} = 2 \times P_{AVG} \qquad \text{(Equation 5)}$$

POWER OF MODULATED RF SIGNALS

In the case of an unmodulated sine wave signal, average power is calculated as in Equation 4. If the load impedance (Z) is known, it is easier to use Equation 6 than to measure the RF current.

$$P_{AVG} = \frac{{V_{RMS}}^2}{Z} \qquad \text{(Equation 6)}$$

Example 1

What is the average power in a 50-Ω load during one complete RF cycle if the RMS voltage is 70 V?

$$P_{AVG} = \frac{70^2}{50} = 98 \text{ W}$$

Only when using CW is a steady sine-wave signal being produced by a transmitter. For AM (including SSB) signals, the waveform varies with time in order to carry information. The peak power output of an AM or SSB transmitter is defined to be the power averaged over a single, complete RF cycle having the greatest amplitude. This is *peak envelope power* or *PEP*. For an unmodulated signal, PEP is equal to average power.

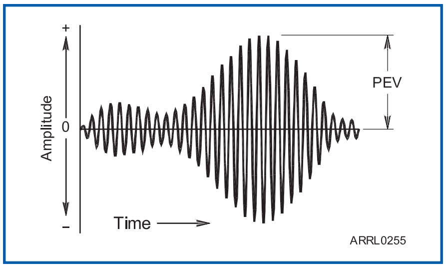

Figure 4 — An amplitude-modulated signal as an example of a complex signal. Peak envelope voltage (PEV) is an important parameter for determining the power of a complex waveform.

For determining PEP of an irregular speech-modulated waveform, the most important parameter is the *peak envelope voltage* (PEV), shown in **Figure 4**. PEV is used when calculating the power in a modulated signal, such as that from an amateur SSB transmitter. To compute the PEP of a modulated waveform, multiply the PEV by 0.707 to obtain the RMS value, square the result and divide by the load resistance.

$$\text{PEP} = \frac{(\text{PEV} \times 0.707)^2}{R_{\text{LOAD}}} \qquad \text{(Equation 7)}$$

Example 2

What is the PEP output power of a transmitter that has a PEV of 100 V across a resistive load of 50 Ω?

$$\text{PEP} = \frac{(100 \times 0.707)^2}{50} = 100 \text{ W}$$