

# Effects of Ground – Graphing Spreadsheets

## *ARRL Antenna Book – 22<sup>nd</sup> Edition*

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### Introduction

On the website supporting the 22nd edition of the *ARRL Antenna Book* ([www.arrl.org/antenna-book](http://www.arrl.org/antenna-book)) there are two Excel (XLSX) spreadsheet files associated with the following graphs in Chapter 3: **Ch3 Fig 6, 7 & 8.xlsx** and **Ch3 Fig 11-22 & 28.xlsx**. In the process of generating the graphs in the book typical values were chosen for such variables as vertical height, frequency of operation, power level, ground characteristics, etc. In the spreadsheets provided on the website, you can choose your own values for the variables and the graphs will change accordingly. The following is a description of those spreadsheets. It is assumed that you have read the material in Chapter 3.

### Ch3 Fig 6, 7 & 8.xlsx

Figures 3.6 and 3.7 describe the skin depth ( $\delta$ ) or depth of penetration in soil. In good conductors the expression for  $\delta$  is relatively simple. However, soil is a complex medium and more complex expression is needed which includes the effect of both conductivity ( $\sigma$ ) and permeability ( $\epsilon$ ).

$$\delta = \left( \frac{\sqrt{2}}{\omega \sqrt{\mu \epsilon}} \right) \sqrt{\sqrt{1 + \left( \frac{\sigma}{\omega \epsilon} \right)^2} - 1} \quad (1)$$

Where:

$\delta$  = skin or penetration depth [m]

$\omega = 2\pi f$

$f$  = frequency [Hz]

$\sigma$  = conductivity [S/m]

$\mu = \mu_r \mu_o$  = permeability [Henry/m]

$\mu_o$  = permeability of a vacuum =  $4\pi 10^{-7}$  [Henry/m]

$\mu_r$  = relative permeability [no units]

$$\epsilon = \epsilon_r \epsilon_0 \text{ [Farad/m]}$$

$$\epsilon_0 = \text{permittivity of a vacuum} = 8.854 \times 10^{-12} \text{ [Farad/m]}$$

$$\epsilon_r = \text{relative permittivity or dielectric constant [no units]}$$

**Figure 3.6** shows the skin depth for several typical soils over a frequency range of 0.1 to 100 MHz. The calculations for figure 3.6 are in sheet 1 of the spreadsheet. Equation (1) is embedded in the cells for columns B-E and rows 6-33.

**Figure 3.7** shows the effect of different values of  $\epsilon_r$  on  $\delta$  for two values of  $\sigma$ . The calculations are performed in columns A-H of sheet 2. In this case, where results for two different values of  $\sigma$  were of interest, the calculation was first done for  $\sigma = 0.001$  [S/m] and the values copied and pasted into columns J-Q. Then the values for  $\sigma$  in columns B-H were changed to 0.01 [S/m] and the values for that copied and pasted into columns R-X. If you wish to alter the graph you need to change the values in columns B-H, rows 5 & 6, and then copy and paste these values into the chart data field in columns J-X. You can also just create a new graph. In this case the graph (fig 3.7) is not live, i.e. not directly affected by the calculations in columns A-H, until you copy and paste the data into columns J-X.

**Figure 3.8** shows the wavelength ( $\lambda$ ) of an electromagnetic wave in several typical soils over the range of 0.1 to 100 MHz. The expression with which this graph is computed is:

$$\lambda = \frac{\lambda_0}{\left[ \epsilon_r^2 + \left( \frac{\sigma}{\omega \epsilon_0} \right)^2 \right]^{1/4}}$$

The calculations for figure 3.8 are in sheet 3.

**ch3 Fig 11-22 & 28.xlsx**

**Figure 3.11** shows the H-field intensity ( $H_\phi$ ) at ground level around the base of a vertical and **Figure 3.13** shows the E-field intensity ( $E_z$ ) around the base of vertical at ground level. The equations expressing these fields are:

$$|H_\phi| = \left( \frac{I_o}{2\pi r \lambda} \right) \left[ \frac{1}{\sin(2\pi h)} \right] \cdot \sqrt{[\sin(2\pi r) \cos(2\pi h) - \sin(2\pi r_2)]^2 + [\cos(2\pi r) \cos(2\pi h) - \cos(2\pi r_2)]^2}$$

[A/m]

$$|E_z| = \left[ \frac{0.2 f_{MHz} I_o}{r \sin(2\pi h)} \right] \cdot \sqrt{\left[ \sin(2\pi r) \cos(2\pi h) - \left( \frac{r}{r_2} \right) \sin(2\pi r_2) \right]^2 + \left[ \cos(2\pi r) \cos(2\pi h) - \left( \frac{r}{r_2} \right) \cos(2\pi r_2) \right]^2}$$

[V/m]

Where:

h=height of the vertical in  $\lambda$

r= distance from the base of the vertical in  $\lambda$

$f_{MHz}$ = frequency in MHz

$$r_2 = \sqrt{r^2 + h^2}$$

$I_o$  = current at the base of the antenna

The computation for the data points of Figures 3.11 and 3.13 is done in sheet 1, columns A-G. In this case, because a number of runs were needed to generate the graph, the values from the computations on sheet 1 have been copied and pasted into sheet 1A, columns A-M. On Figure 3.13 there is a dashed line indicating the value of  $H_\phi$  at a constant physical distance of 10.7 m. These data points are in sheet 1A, columns O-R.

The power loss in the soil is a function of both  $H_\phi$  and  $E_z$  :

$$PH' = Re \cdot |H_\phi|^2 \quad [W/m^2] \text{ and } PE' = Ge \cdot |E_z|^2 \quad [W/m^2]$$

Where:

$$Re = \sqrt{\left\{ \frac{\mu_o}{2\varepsilon \left[ 1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2 \right] \right\} \left\{ 1 + \sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2} \right\}} \quad [\Omega]$$

$$Ge = \left( \frac{\sqrt{2}\sigma}{2\omega\epsilon_r^2\sqrt{\mu\epsilon}} \right) \left[ \frac{1}{\sqrt{1 + \left( \frac{\sigma}{\omega\epsilon} \right)^2} - 1} \right] \left[ \frac{1}{1 + \left( \frac{\sigma}{\omega\epsilon} \right)^2} \right] \text{ [S]}$$

PH' is the power dissipated in one square meter of soil for a given value of  $H_\phi$  and PE' is the power dissipated in one square meter of soil for a given value of  $E_z$ . To compute the total power loss within a radius  $r$  of the base of the vertical, the area around the vertical is divided up into small rings with a width  $\Delta r$  and a radius  $r$ . The loss in the individual rings ( $P_a$  or  $P_b$ ) is:

$$P_a = 2\pi r \Delta r PH' \text{ [W]} \text{ and } P_b = 2\pi r \Delta r PE' \text{ [W]}$$

The next step is add up the power in each ring out to the maximum value for  $r$ . This gives the value for the ground loss, PH and PE. The total ground loss is PH+PE.

The calculations for PH are given on sheet 1 and on sheet 2 for PE. The values for particular choices of variables have been copied and pasted to sheets 1A and 2A which are the data for Figures 3.11, 3.112, 3.13 and 3.14.

Some of the calculations on sheets 1 and 2 have been copied and pasted into sheet 3 which provides the total power loss summary shown in Figure 3.16 and the comparison between E-field and H-field power losses in Figure 3.17.

The next step is to calculate the effective ground resistance ( $R_g$ ):

$$R_g = \frac{PH+PE}{I_0^2} \text{ [\Omega]}$$

The calculations for  $R_g$  are on sheet 3 and displayed in Figures 3.19, 3.20, 3.21 and 3.22.

The voltage from radials to ground shown in Figure 3.28 was derived from near-field modeling using EZNEC. Where do the numbers in Figure 3.28 come from? One of the features of most NEC software is the ability to calculate the electric (E) and magnetic (H) field intensities close to the antenna, i.e. in the near-field. If you know the values for the E-field for a path between two points, say ground and some point on a radial, you can calculate the potential difference (i.e. the voltage) between the two points.

$$V = \int_a^b \mathbf{E} \cdot d\mathbf{l} \text{ [V]}$$

Where  $\mathbf{E}$  is the vector E-field and  $\mathbf{l}$  is the increment of distance along the path of integration in vector form.

Fortunately this integral is path independent so by being a little clever in our choice of the path, we don't have to deal with vector calculus! NEC can give us the values for  $E$  in the  $z$  direction (i.e. from ground vertically up to the radial) from some point on the ground (a) directly below a point on a radial (b). We can divide the path into short intervals and assume that  $E$  is constant over a particular interval to get the voltage across that interval. We can then sum these voltages to get the voltage between a point on the radial and the corresponding point on the ground. This is what's done on sheet 4 for four, twelve and thirty two radials. The summation of the voltages across each small interval is done in rows 50, 100 and 150.

The values for  $|E_z|$  in columns C-X were obtained from an EZNEC model. These values can be replaced with values from another antenna model.